ON A PERIODIC CONTACT PROBLEM FOR A HALF-PLANE WITH ELASTIC COVERINGS

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The problem mentioned in the title is reduced to an integro-differential equation of Prandtl type with a Hilbert kernel for which an approximate solution has been given in [1].

The method of orthogonal polynomials [2] is utilized herein for the approximate solution of this equation (which is also encountered in other branches of mathematical physics). It can be stated that the method elucidated herein leads to the results more simply and rapidly. This permits giving a numerical realization of the proposed method in a comparatively small amount of calculations.

1. Let elastic coverings with cross-sectional area F be welded (glued) to an infinite plate (half-plane) of thickness h at finite segments from its boundaries

$$[-a + 2nl,\ a + 2nl], \quad (l > a,\ n = 0, \pm 1, \ldots)$$

As has been shown in [1], seeking the contact stress $\tau(x')$ originating along the line of contact between the covering and the half-plane is associated with the solution of the following equation:

$$\varphi(\xi) + \frac{1}{2\pi\lambda}\int_{-\alpha}^{\alpha} \operatorname{ctg}\frac{\xi - \eta}{2}\,\varphi'(\eta)\,d\eta = g(\xi) \qquad (|\alpha| < \pi) \tag{1.1}$$

$$\left(\int_{-\alpha}^{\xi} t(\eta)\,d\eta = \varphi(\xi),\ \lambda = \frac{E_2 lh}{2\pi E_1 F},\ \alpha = \frac{\pi a}{l},\ \tau(a\xi/\alpha) = t(\xi)\right)$$

Here E_1, E_2 are, respectively, the elastic modulus of the covering and the half-plane. The required solution $\tau(x')$ should be subject to one of the equilibrium conditions

$$h\int_{-a}^{a} \tau(x')\,dx' = R, \qquad \int_{-\alpha}^{\alpha} t(\xi)\,d\xi = \frac{\alpha R}{ah} \tag{1.2}$$

Here R denotes the resultant of all the forces applied to the covering.

Let us put

$$\int_{-\alpha}^{\xi} t(\eta)\,d\eta = \frac{1}{2}\int_{-\alpha}^{\alpha} \operatorname{sign}(\xi - \eta)\,t(\eta)\,d\eta + \frac{\alpha R}{2ah}$$

Taking account of this latter expression (1.1) can be written as

$$\int_{-\alpha}^{\alpha}\left[\frac{1}{2}\operatorname{sign}(\xi - \eta) + \frac{1}{2\pi\lambda}\operatorname{ctg}\frac{\xi - \eta}{2}\right] t(\eta)\,d\eta = f(\xi) \qquad (|\alpha| < \pi) \tag{1.3}$$

$$(f(\xi) = g(\xi) - \alpha R/2ah)$$

2. Formulas from the handbook [3] are often utilized below. For brevity, let us indicate just the number of the formula without repeating the source each time.

The proposed approximate method of solving Eqs. (1.3) or (1.1) is based on the following relationships for the Chebyshev polynomials:

$$\int_{-\alpha}^{\alpha} \operatorname{ctg}\frac{\xi-\eta}{2}\frac{T_m(\operatorname{tg}{}^1/_2\eta\operatorname{ctg}{}^1/_2\alpha)}{\sqrt{2\cos\eta-2\cos\alpha}}\frac{d\eta}{\cos{}^1/_2\eta} = J_{1,m}(\xi) \tag{2.1}$$

$$J_{1,0}(\xi) = -\pi\sec{}^1/_2\alpha\operatorname{tg}{}^1/_2\xi \qquad (m=0)$$

$$J_{1,m}(\xi) = -\pi\csc{}^1/_2\alpha(\sec{}^1/_2\xi)^2 U_{m-1}(\operatorname{tg}{}^1/_2\xi\operatorname{ctg}{}^1/_2\alpha) \qquad (m=1,2,\ldots)$$

$$\frac{1}{2}\int_{-\alpha}^{\alpha}\operatorname{sign}(\xi-\eta)\frac{T_m(\operatorname{tg}{}^1/_2\eta\operatorname{ctg}{}^1/_2\alpha)}{\sqrt{2\cos\eta-2\cos\alpha}}\frac{d\eta}{\cos{}^1/_2\eta} = J_{2,m}(\xi) \tag{2.2}$$

$$J_{2,0}(\xi) = \sec{}^1/_2\alpha\arcsin(\operatorname{tg}{}^1/_2\xi\operatorname{ctg}{}^1/_2\alpha) \qquad (m=0)$$

$$J_{2,m}(\xi) = -(m\sin\alpha\cos{}^1/_2\xi)^{-1}\sqrt{2\cos\xi-2\cos\alpha}\,U_{m-1}(\operatorname{tg}{}^1/_2\xi\operatorname{ctg}{}^1/_2\alpha) \qquad (m=1,2,\ldots)$$

Here $T_m(x)$, $U_m(x)$ are, respectively, the Chebyshev polynomials of the first and second kinds.

To prove (2.1), let us start from the known relationship

$$\int_{-1}^{1}\ln|x-y|\frac{T_m(y)\,dy}{\sqrt{1-y^2}} = \begin{cases}-\pi\ln 2 & (m=0)\\ -\pi m^{-1}T_m(x) & (m=1,2,\ldots)\end{cases} \tag{2.3}$$

Making the substitution $x = \operatorname{tg}{}^1/_2\xi\operatorname{ctg}{}^1/_2\alpha$, $y = \operatorname{tg}{}^1/_2\eta\operatorname{ctg}{}^1/_2\alpha$ and utilizing the known properties of trigonometric functions, we will have

$$\int_{-\alpha}^{\alpha}\ln\frac{\left|\sin\frac{\xi-\eta}{2}\right|}{\cos{}^1/_2\xi\cos{}^1/_2\eta}\frac{T_m(\operatorname{tg}{}^1/_2\eta\operatorname{ctg}{}^1/_2\alpha)}{\sqrt{2\cos\eta-2\cos\alpha}}\frac{d\eta}{\cos{}^1/_2\eta} = J_{3,m}(\xi) \tag{2.4}$$

$$J_{3,0}(\xi) = -\pi\sec{}^1/_2\alpha\ln(2\operatorname{ctg}{}^1/_2\alpha) \qquad (m=0)$$

$$J_{3m}(\xi) = -\pi m^{-1}\sec{}^1/_2\alpha\, T_m(\operatorname{tg}{}^1/_2\xi\operatorname{ctg}{}^1/_2\alpha) \qquad (m=1,2,\ldots)$$

Differentiating (*) the last equation we obtain (2.1).

The same substitution in the formula [4]

$$\frac{1}{2}\int_{-1}^{1}\operatorname{sign}(x-y)\frac{T_m(y)\,dy}{\sqrt{1-y^2}} = \begin{cases}\arcsin x & (m=0)\\ -m^{-1}\sqrt{1-x^2}\quad U_{m-1}(x) & (m=1,2,\ldots)\end{cases}$$

results in the relationship (2.2).

3. Let us seek the solution of (1.3) in the form

$$t(\xi) = \frac{\sec{}^1/_2\xi}{\sqrt{2\cos\xi-2\cos\alpha}}\sum_{m=0}^{\infty}X_m T_m(\operatorname{tg}{}^1/_2\xi\operatorname{ctg}{}^1/_2\alpha) \tag{3.1}$$

Let us substitute (3.1) into (1.3) and let us utilize the relationships (2.1) and (2.2).

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*) The relationship (2.1) can also be obtained directly from formula 7.344(1) by means of the same substitution. We start here from (2.3) so as to obtain the spectral relationship (2.4), which is itself of independent interest. In particular, it permits giving the solution of the integral equation of the periodic contact problem [5, 6] in the form of an infinite series of Chebyshev polynomials. Such a form of the solution will sometimes be preferred as compared with the solutions in the form of quadratures obtained in [5, 6].

Then let us multiply the equality obtained by

$$\sqrt{2\cos\xi - 2\cos\alpha} \times \sec {}^1/_2\xi\, U_k\, (\mathrm{tg}\, {}^1/_2\xi\, \mathrm{ctg}\, {}^1/_2\alpha)$$

and let us integrate with respect to ξ between the limits $(-\alpha, \alpha)$. Utilization of the orthogonality properties of the Chebyshev polynomials (rephrased for the variable ξ) results in the following infinite system of equations in the unknown coefficients X_m ($m = 1, 2, \ldots$):

$$(\pi / 4\lambda)\, \mathrm{ctg}\, {}^1/_2\alpha X_{k+1} + \sum_{m=1}^{\infty} B_{m-1,\,k} X_m = C_k X_0 - \cos {}^1/_2\alpha b_k \quad (k = 0, 1, \ldots) \tag{3.2}$$

Here

$$B_{m-1,\,k} = \frac{1}{m} \int_{-1}^{1} \frac{(1 - x^2)\, U_{m-1}(x)\, U_k(x)}{1 + x^2 (\mathrm{tg}\, {}^1/_2\alpha)^2} dx \quad \begin{pmatrix} m = 1, 2, \ldots \\ k = 0, 1, \ldots \end{pmatrix} \tag{3.3}$$

$$C_k = \int_{-1}^{1} \frac{[\arcsin x - (x / 2\lambda)\, \mathrm{tg}\, {}^1/_2\alpha]\, \sqrt{1 - x^2} U_k(x)}{1 + x^2 (\mathrm{tg}\, {}^1/_2\alpha)^2} dx \quad (k = 0, 1, \ldots) \tag{3.4}$$

$$b_k = \int_{-1}^{1} \frac{p(x) \sqrt{1 - x^2} U_k(x)}{1 + x^2 (\mathrm{tg}\, {}^1/_2\alpha)^2} dx \quad (k = 0, 1, \ldots) \tag{3.5}$$

$$p(x) = f\, [\mathrm{arctg}\, (x\, \mathrm{tg}\, {}^1/_2\alpha)]$$

The new variable $x = x' / a = \mathrm{tg}{}^1/_2\xi \mathrm{ctg}{}^1/_2\alpha$ has been introduced in these latter expressions, for convenience.

The coefficient X_0 is determined from the second equation of (1.2). Substituting (3.1) into the mentioned equation and utilizing the orthogonality condition of Chebyshev polynomials, we obtain $X_0 = (R / lh) \cos{}^1/_2\alpha$.

Let us use the expansion

$$\frac{1}{1 + x^2 (\mathrm{tg}\, {}^1/_2\alpha)^2} = 2\, \mathrm{ctg}\, {}^1/_2\alpha \sum_{n=0}^{\infty} (-1)^n (\mathrm{tg}\, {}^1/_4\alpha)^{2n+1}\, U_{2n}(x) \quad (|\alpha| < \pi,\ |x| \leqslant 1) \tag{3.6}$$

for a practical calculation of the coefficients $B_{m-1,k}$ and C_k.

To verify this latter, both sides should be multiplied by $\sqrt{1 - x^2} U_{2k}(x)$ and integrated with respect to x between the limits $(-1, 1)$. To evaluate the integral obtained in the left side, the function $[1 + x^2 (\mathrm{tg}\, {}^1/_2\alpha)^2]^{-1}$ should be replaced by its integral representation given by 3.893(2), the order of integration should be changed and 3.715(18) and 6.611(1) used.

Substituting (3.6) into (3.3) and (3.4) and using the formulas

$$\int_{-1}^{1} (1 - x^2)\, U_m(x)\, U_k(x)\, U_n(x)\, dx = -\frac{4(m+1)(k+1)(n+1)\,[\cos {}^1/_2 (m+k+n)\pi]^2}{[(m+1)^2 - (k-n)^2]\,[(m+1)^2 - (k+n+2)^2]} \tag{3.7}$$

$$(m, k, n = 0, 1, \ldots)$$

$$\int_{-1}^{1} \arcsin x \sqrt{1 - x^2}\, U_k(x)\, U_n(x)\, dx = \frac{4(k+1)(n+1)\,[\sin {}^1/_2 (k+n)\pi]^2}{(k-n)^2 (k+n+2)^2} \tag{3.8}$$

$$(k, n = 0, 1, \ldots)$$

$$\int_{-1}^{1} \frac{x \sqrt{1 - x^2}\, U_k(x)}{1 + x^2 (\mathrm{tg}\, {}^1/_2\alpha)^2} dx = \pi \sin {}^1/_2 k\pi\, (\mathrm{ctg}\, {}^1/_2\alpha)^2 (\mathrm{tg}\, {}^1/_4\alpha)^{k+1} \quad (k = 0, 1, \ldots) \tag{3.9}$$

we arrive at the following expressions for the coefficients $B_{m-1,k}$ and C_k:

$$B_{m-1,k} = -8\,(k+1)\,\mathrm{ctg}\,{}^1/_2\,\alpha \sum_{n=0}^{\infty} \frac{(-1)^n (2n+1)(\mathrm{tg}\,{}^1/_4\,\alpha)^{2n+1}[\sin {}^1/_2\,(m+k)\,\pi]^2}{[m^2-(2n-k)^2]\,[m^2-(2n+k+2)^2]} \tag{3.10}$$

$$(m, k = 0, 1, \ldots)$$

$$C_k = -B_{-1,k} - (\pi / 2\lambda) \sin {}^1/_2\, k\pi\, \mathrm{ctg}\,{}^1/_2\,\alpha\, (\mathrm{tg}\,{}^1/_4\,\alpha)^{k+1} \quad (k = 0, 1, \ldots) \tag{3.11}$$

To verify (3.7)–(3.9), the substitution $x = \cos\theta$ should be made and it should be taken into account that $U_m(\cos\theta) = \sin[(m+1)\theta]\cos\theta$. Moreover, its integral representation already mentioned above should be substituted for the function

$$[1 + x^2 (\mathrm{tg}\,{}^1/_2\alpha)^2]^{-1}$$

and the order of integration should be changed.

Because of the rapid convergence of the series (3.10), evaluation of the coefficients $B_{m-1,k}$ does not involve much work.

Let us represent the function $\tau(x')$ as $\tau(x') = \tau^+(x') + \tau^-(x')$, where $\tau^+(x')$ and $\tau^-(x')$ are the even and odd parts of the function $\tau(x')$, respectively. It is seen from the structure of the terms $B_{m-1,k}$ that the system (3.2) decomposes into two, one of which contains the unknowns X_m with even subscripts (the even problem, $\tau(x') = \tau^+(x')$), and the other with the odd subscripts (the odd problem, $\tau(x') = \tau^-(x')$). In the first case the loading acting on the covering should be skew-symmetric ($p(x) = p^-(x)$), and in the second symmetric ($p(x) = p^+(x)$). To obtain the solution of the even problem m should be replaced by $2m$ and k by $2k+1$ in (3.2)–(3.5), (3.10) and (3.11). The odd case is obtained from the same formulas by replacing m by $2m+1$ and k by $2k$.

Let us examine two particular cases of loading the coverings. Let the covering be loaded by two forces of magnitude $0.5P$ each, applied to the covering endfaces and directed to one side (from left to right). In this case

$$p(x) \equiv 0,\ \tau(x') = \tau^+(x'),\ b_{2k} = b_{2k+1} = 0\ (k = 0, 1, \ldots),\ X_0 = (P / lh)\cos {}^1/_2\alpha.$$

From the equilibrium condition for an element of the covering it is easy to obtain a formula to compute the normal stresses in the section x' of the covering. The corresponding formula for the variable $x = x' / a$ is

$$\sigma^-(ax) = (lh/\pi F) \sec {}^1/_2\,\alpha \left[X_0 \arcsin x - \sqrt{1-x^2} \sum_{m=1}^{\infty} (2m)^{-1} X_{2m} U_{2m-1}(x)\right]$$

Let us present a formula to compute the contact stresses $\tau(x')$ written in the variable $x = x' / a$

$$\tau^+(ax) = \frac{1}{2\sin {}^1/_2\,\alpha} \frac{1 + x^2 (\mathrm{tg}\,{}^1/_2\,\alpha)^2}{\sqrt{1-x^2}} \sum_{m=0}^{\infty} X_{2m} T_{2m}(x) \tag{3.12}$$

When the forces applied to the covering endfaces stretch it (the odd problem), we have the analogous results $\quad p(x) = p^+(x) = \pi P / 2hl, \quad X_0 = 0$

$$b_{2k} = 0, \quad b_{2k+1} = (\pi^2 P / 2hl)(-1)^k \cos {}^1/_2\alpha\, \mathrm{ctg}\,{}^1/_2\alpha\, (\mathrm{tg}\,{}^1/_4\alpha)^{2k+1} \quad (k = 0, 1, \ldots)$$

$$\sigma^+(ax) = (P/F)\left[0.5 - (hl / \pi P) \sec {}^1/_2\,\alpha \sqrt{1-x^2} \sum_{m=0}^{\infty} (2m+1)^{-1} X_{2m+1} U_{2m}(x)\right]$$

The contact stresses $\tau^-(ax)$ can be computed by means of (3.12) if $2m$ is first replaced by $2m+1$.

The relationship

$$\int_{-1}^{1} \frac{\sqrt{1-x^2}U_k(x)}{1+x^2(\operatorname{tg} {}^1/_2\alpha)^2}\,dx = \pi \cos {}^1/_2 k\pi \operatorname{ctg}^{1/2}\alpha\, (\operatorname{tg} {}^1/_4\alpha)^{k+1} \qquad (|\alpha| < \pi,\ k = 0, 1, ..)$$

which is verified exactly as is (3.9), was used in computing the coefficients b_k.

4. Let us investigate the system (3.2). Let us first find for which values of the parameter λ will the system be completely regular. To this end let us estimate the sum

$$S_k = (4\lambda/\pi)\operatorname{tg} {}^1/_2\alpha \sum_{m=1}^{\infty} |B_{m-1,k}| \qquad (k = 0, 1, ..)$$

Let us put

$$B_{m-1,k} = \frac{1}{m} D_{m-1,k}, \qquad D_{m-1,k} = \int_{-1}^{1} \frac{(1-x^2)\,U_{m-1}(x)\,U_k(x)}{1+x^2(\operatorname{tg} {}^1/_2\alpha)^2}\,dx$$

$$(m = 1,2,\ldots,\ k = 0,1,\ldots)$$

The Buniakowski inequality for the sums yields

$$S_k^2 \leqslant \left(\frac{4\lambda \operatorname{tg} {}^1/_2\alpha}{\pi}\right)^2 \sum_{m=1}^{\infty} \frac{1}{m^2} \sum_{m=1}^{\infty} D_{m-1,k}^2 \tag{4.1}$$

Let us examine the function

$$\omega(x) = \frac{\pi}{2} \frac{\sqrt{1-x^2}\,U_k(x)}{1+x^2(\operatorname{tg} {}^1/_2\alpha)^2}$$

The coefficients of its expansion in a series of polynomials $U_{m-1}(x)$ $(m = 1,2,\ldots)$ will be $D_{m-1,k}$. The completeness (closure) condition of this system is

$$\sum_{m=1}^{\infty} D_{m-1,k}^2 = \frac{\pi^2}{4} \int_{-1}^{1} \frac{(1-x^2)^{3/2}\,U_k^2(x)}{[1+x^2(\operatorname{tg} {}^1/_2\alpha)^2]^2}\,dx$$

Since $U_k^2(x) \leqslant 1$, $(|x| \leqslant 1)$, it is then easy to estimate this last integral, and then

$$\sum_{m=1}^{\infty} D_{m-1,k}^2 \leqslant \frac{3\pi^2}{16}$$

Summing the first series in (4.1), and taking account of the above, we obtain

$$S_k^2 \leqslant {}^1/_2\pi^2\lambda^2(\operatorname{tg} {}^1/_2\alpha)^2, \quad S_k \leqslant {}^1/_2\sqrt{2}\,\pi\lambda \operatorname{tg} {}^1/_2\alpha \tag{4.2}$$

In order for the system (3.2) to be completely regular, it is sufficient that the inequality $S_k \leqslant q < 1$ be satisfied for all k $(k = 0, 1,\ldots)$. Taking account of (4.2), this condition becomes

$$\lambda < \sqrt{2}\,\pi^{-1}\operatorname{ctg} {}^1/_2\alpha \tag{4.3}$$

The condition obtained is considerably broader than the condition $\lambda < {}^1/_{25}\sin\alpha$ obtained in [1].

Now, let us show that for any λ $(0 \leqslant \lambda < \infty)$ the system (3.2) is quasi-regular, i.e. let us show that $\lim S_k = 0$ as $k \to \infty$. Since the system (3.2) decomposes into two systems, let us then investigate them separately. Let us start with the even case. In this case it is necessary to estimate the sum

$$S_{2k+1} = (4\lambda/\pi)\operatorname{tg} {}^1/_2\alpha \sum_{m=1}^{\infty} |B_{2m-1,2k+1}| \tag{4.4}$$

Let us first note that the estimate

$$|B_{2m-1,\,2k+1}| \leqslant 1/(k+1) + 2\pi (\sec {}^1/_2\, \alpha)^2 / (4k+3)(4k+5) \qquad (m = k+1,\ k = 0, 1, \ldots)$$

$$|B_{2m-1,\,2k+1}| \leqslant \frac{\pi (\sec {}^1/_2\, \alpha)^2}{8m} \Big(\frac{1}{|2m-2k-2|+1} + \frac{1}{|2m-2k-2|-1} + \frac{1}{2m+2k+3} + \frac{1}{2m+2k+1} \Big) \tag{4.5}$$

$$(m = 1, 2, \ldots;\ k = 0, 1, \ldots;\ m \neq k+1)$$

can be given for the coefficients $B_{2m-1,\,2k+1}$.

To prove this latter inequality, the identity

$$2(1-x^2)\, U_{2m-1}(x)\, U_{2k+1}(x) = T_{2m-2k-2}(x) - T_{2m+2k+2}(x)$$

should be utilized in (3.3), the integral should be separated into two, and it should be taken into account that

$$\int_{-1}^{1} \frac{T_{2m}(x)\, dx}{1 + x^2 (\operatorname{tg} {}^1/_2\, \alpha)^2} \leqslant \frac{\pi}{2(\cos {}^1/_2\, \alpha)^2} \Big(\frac{1}{2m+1} + \frac{1}{2m-1} \Big) \qquad (|\alpha| < \pi,\ m = 1, 2, \ldots) \tag{4.6}$$

To obtain the last estimate, $x = \cos\theta$ should be substituted in the integral, and it should be taken into account that $T_m(\cos\theta) = \cos m\theta$. Then utilizing 1.314(1), and integrating by parts, we obtain the estimate (4.6). Substituting (4.5) into (4.4) and summing the series obtained by using 0.244(1) we obtain

$$S_{2k+1} \leqslant \frac{4\lambda \operatorname{tg} {}^1/_2\, \alpha}{\pi} \Big\{ \Big[\frac{\psi(k + {}^5/_2) + \psi(k+2) - 2\psi({}^1/_2) + \psi(-k - {}^1/_2) + 2C}{2k+3} + \frac{\psi(k + {}^3/_2) + 2\psi(k+2) - 2\psi({}^3/_2) + \psi(-k + {}^1/_2) + 2C}{2k+1} + \frac{8}{(4k+3)(4k+5)} \Big] \frac{\pi}{8(\cos {}^1/_2\, \alpha)^2} + \frac{1}{k+1} \Big\} \tag{4.7}$$

Here $\psi(x)$ is the Euler psi-function, and C the Euler constant.

Taking account of the behavior of the function $\psi(x)$ as $x \to \infty$, the right side of (4.7) can be made arbitrarily small for any λ for sufficiently large k, and therefore, the system (3.2) (in the even case) is quasi-regular for $0 \leqslant \lambda < \infty$.

An analogous result is obtained for the odd system. The condition of complete regularity of the system (3.2) can be indeed obtained from the inequality (4.7), however the result (4.3) obtained above is more accurate.

5. Computations of $\sigma(ax)$ and $\tau(ax)$ were performed by the formulas obtained above for the case when the covering was loaded on the endfaces, for different values of the parameters λ and α. The values of $\sigma(ax)$ and $\tau(ax)$ were computed for successively increasing numbers of the approximation m. If $\sigma(ax)$ or $\tau(ax)$ in the $(m+1)$-th approximation differed from $\sigma(ax)$ or $\tau(ax)$ in the mth approximation by less than 5%, then the calculation process was stopped.

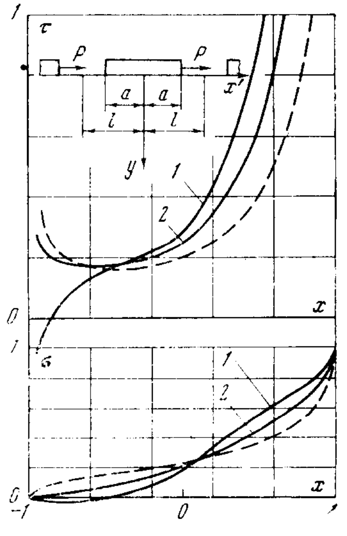

Fig. 1

The quantity of approximations needed depends essentially on the parameter α and to a lesser degree on λ. For example, for all λ not more than twice larger than the domain of regularity determined by means of (4.3), three approximations are sufficient to compute $\tau(ax)$ for $\alpha = {}^1/_4\pi$, and two to compute $\sigma(ax)$. For $\alpha = {}^3/_4\pi$ six approximations must be taken to compute $\tau(ax)$, and four to compute $\sigma(ax)$.

The explanation of the mutual influence of the coverings is quite important. This influence evidently depends primarily on α. However, it has been clarified that it also depends on the parameter $\mu = \lambda / \alpha = E_2 ah / 2E_1 F$. Calculations were performed for different values of the parameters μ and α when the covering is loaded along the end-faces (where as above, the problem was separated into even and odd). The results of the calculations are presented in the table where the maximum deviations of the stresses, in absolute value, from those in the case when there is just one isolated covering, are indicated in percentage. Here, not the stresses themselves were compared in the case of the tangential compact stresses, but their values multiplied by a function with a singularity. The mentioned deviations were observed mainly in the neighborhood of the ends of the covering. The quantity of approximations m in the stress computation was taken so that $\sigma(ax)$ or $\tau(ax)$ in the $(m+1)$-th approximation would not differ from the $\sigma(ax)$ or $\tau(ax)$ in the mth approximation by more than several units in the third significant figure.

α	σ^+		σ^-		τ^+		τ^-	
	$\mu = 1$	$\mu = 5$	$\mu = 1$	$\mu = 5$	$\mu = 1$	$\mu = 5$	$\mu = 1$	$\mu = 5$
${}^1/_5\pi$	23	60	231	265	90	240	220	335
${}^2/_3\pi$	14	33	103	115	51	118	101	144
${}^1/_2\pi$	7	15	44	48	43	50	41	57
${}^1/_3\pi$	3	6	16	18	12	19	16	21

Presented in Fig. 1 are graphs of $\sigma(ax)$ and $\tau(ax)$ for $\mu = 1$ (for the computation scheme shown there). Curves *1* and *2* refer to the cases $\alpha = {}^4/_5\pi$ and $\alpha = {}^2/_3\pi$, respectively. The dashes show the corresponding stresses for the case of one isolated covering. The values of $\sigma(ax)$ presented in the graph should be multiplied by P / F, and those of $\tau(ax)$ by P / ah. The calculations were also performed for the parameters $\alpha = {}^1/_2\pi$; ${}^2/_5\pi$ and ${}^1/_3\pi$. The corresponding curves (which are not shown in the graph in order not to complicate it inordinately) are located between curves *2* and the dashes. For $\alpha = {}^1/_3\pi$ these curves practically coincide with the dashes (the maximum deviation does not exceed 15%).

As is seen from Fig. 1, the presence of adjacent coverings can alter the stress diagram (curve *1*) qualitatively as well as quantitatively.

In conclusion, let us note that the problem considered here is equivalent to the contact problem for a half-strip $(-l \leqslant x \leqslant l,\ 0 \leqslant y < \infty)$ when an elastic covering is glued at the segment $(-a \leqslant x \leqslant a)$ to its finite face, and the vertical displacement and normal stress are zero on the semiinfinite faces.

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EFFECT OF SHEARING FORCE AND TILTING MOMENT ON A CYLINDRICAL PUNCH ATTACHED TO A TRANSVERSELY ISOTROPIC HALF-SPACE

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The effect of shearing force and tilting moment on a rigid punch, circular in plan view, and attached to a transversely isotropic half-space is considered. The distribution of shear and normal stresses under the punch is derived. Formulas are obtained which relate the angular and linear displacements of the punch to the magnitude of the shearing force and tilting moment. Let us point out that the nonaxisymmetric case of a cylindrical punch attached to an ispotropic half-space is solved in [1].

1. Let us consider a circular punch with radius a attached to a transversely isotropic half-space $z \geqslant 0$. Let the punch be subjected to a shearing force T, directed along the x-axis, and a tilting moment M. Without restricting the general nature of the problem, we may assume that the moment is directed along the y-axis. Our problem is to determine the stresses under the punch as well as the two displacements of the punch: translation u_0 and rotation δ.

Let us introduce complex displacements $u = u_x + iu_y$ and complex shear stresses $\tau = \tau_{zx} + i\tau_{yz}$ both in the plane $z = 0$. Making use of the results obtained in [2] we can write the expressions that determine the displacements of a point, having cylindrical coordinates $(\rho, \varphi, 0)$ under the action of a concentrated force, with projections P_x, P_y, P_z, applied at point $(\rho_0, \varphi_0, 0)$

$$u = \frac{1}{2R}\left\{G_1(P_x + iP_y) + G_2(P_x - iP_y)\frac{\rho e^{i\varphi} - \rho_0 e^{i\varphi_0}}{\rho e^{-i\varphi} - \rho_0 e^{-i\varphi_0}}\right\} - \frac{P_z H\alpha}{\rho e^{-i\varphi} - \rho_0 e^{-i\varphi_0}}$$

$$w = H\alpha \operatorname{Re}\left\{\frac{P_x + iP_y}{\rho e^{i\varphi} - \rho_0 e^{i\varphi_0}}\right\} + \frac{P_z H}{R} \qquad (1.1)$$

Here